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Advanced Composite Materials
Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/tacm20

A Mathematical Analysis of Thermoelastic Characteristics of a Rotating Circular Disk with an FGM Coating at the Outer Surface

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Version of record first published: 02 Apr 2012.

To cite this article: A. M. Afsar , J. Go & J. I. Song (2010): A Mathematical Analysis of Thermoelastic Characteristics of a Rotating Circular Disk with an FGM Coating at the Outer Surface, Advanced Composite Materials, 19:3, 269-288

To link to this article: http://dx.doi.org/10.1163/092430410X490482

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Advanced Composite Materials 19 (2010) 269–288
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A Mathematical Analysis of Thermoelastic Characteristics of a Rotating Circular Disk with an FGM Coating at the Outer Surface

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Received 5 February 2009; accepted 24 August 2009

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Abstract

A thin circular rotating disk having a concentric hole and a functionally graded material (FGM) coating at the outer surface is considered with a view to analyzing the thermoelastic characteristics due to a thermal load and rotation of the disk. The FGM coating is assumed to have exponentially varying Young's modulus, coefficient of thermal expansion (CTE), and density in the radial direction of the disk. The Poisson's ratio is assumed to be constant throughout the disk. The incompatible eigenstrain developed in the disk owing to the nonuniform CTE and variation of temperature is taken into consideration. Using the two-dimensional thermoelastic theories, the two-dimensional plane stress axisymmetric problem is formulated as a second order differential equation. A finite element model is developed using the variational approach and Ritz method to obtain the numerical solution of the differential equation. The validity of the finite element model is justified for a rotating circular disk of homogeneous material by comparing the finite element results with analytical solution obtained by Timoshenko. Then the finite element model is applied to the problem of an Al disk with an Al_2O_3/Al FGM coating at its outer surface. The numerical results of the thermoelastic field demonstrate that the temperature distribution profile, angular speed of the disk, and FGM coating thickness are the crucial factors to be considered in controlling the thermoelastic characteristics of a rotating disk with an FGM coating.

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Keywords

Circular disk, functionally graded material, coating, finite element method, thermoelasticity

1. Introduction

Functionally graded materials (FGMs) are a special group of nonhomogeneous composites which consist of two or more distinct material phases with continuously varying material distribution of the constituent materials. Thus, the mechanical and thermal properties of FGMs along with their microstructures have spatial varia-

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Edited by the KSCM

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tion which is the key distinguishing feature of these materials. The concept of these materials was introduced in an attempt to develop thermal barrier coatings for the propulsion system of space planes [1]. Usually, these materials are made of ceramics and metals to resist severe environmental effects, such as wear, corrosion, and large temperature gradient in one hand, and ensure toughness and thermal conductivity on the other hand. Although, the FGMs were developed with the original objective for application in a high temperature gradient field, these materials have now been promising candidates for various engineering structural applications. Among different structural elements, FGM beams [2–4], plates [5–7] and cylinders [8–11] have been studied extensively to explore their performance under various mechanical and thermal loading conditions. A laminated functionally graded Timoshenko beam under the action of thermal load was considered by Xiang and Yang [2] to analyze the free and forced vibrations for variable thickness. Li *et al.* [3] investigated the post-buckling behaviors of an FGM Timoshenko beam under thermal loads. The equivalent homogeneous beam was considered for the analysis of buckling and fracture behaviors of cracked FGM beams by Upadhyay and Simha [4]. The buckling behaviors of FGM plates were studied by Feldman and Aboudi [5] and Chen and Liew [6] for uniaxial load and nonlinearly distributed in-plane edge loads, respectively. Afsar *et al.* [7] examined the effects of nonhomogeneous parameter on the elastic field in an FGM rectangular plate subjected to a biaxial tensile load. FGM circular cylinders were considered by Obata and Noda [8] and Liew *et al.* [9] to analyze the thermal stresses. FGM circular cylinders were also considered by Afsar and his co-workers [10, 11] for the analysis of brittle fracture characteristics by taking into account the effect of incompatibility of eigenstrain developed in the cylinder due to nonuniform CTE as a result of temperature change. Elastoplastic response of an FGM hollow shaft [12] and stress and displacement of an FGM sandwich solid disk [13] were investigated by taking the rotation of the shaft and disk into account.

In the studies mentioned above, attention was focused mainly on the structural performance of FGMs. To investigate the functional performance of these materials, FGMs are widely explored for thermal barrier [14–16] and tribological [17, 18] coatings. However, the analysis of FGM performance as cutting and grinding tools has received only a little attention so far. Cho and Park [19] investigated the thermoelastic characteristics of functionally graded lathe cutting tools composed of Cr–Mo steel and ceramic tip. They demonstrated that an added FGM layer between the steel shank and the ceramic tip relaxes the thermo-mechanical stress concentration. In our previous study [20], we have demonstrated that a circular FGM cutter or grinding disk can be designed with better thermoelastic characteristics if certain parameters, namely, temperature distribution, angular speed, radial thickness, and outer surface temperature, are controlled properly. For the purpose of analyzing the thermoelastic characteristics, a finite element model was developed. However, the FGM disk considered in our previous study [20] has a material gradation throughout the entire radial thickness, i.e., from the inner surface to the outer surface of the

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disk. In practice, it is not feasible to manufacture a disk with graded material distribution throughout the entire radial thickness due to the manufacturing limitation as well as the huge cost involvement. Also, from engineering points of view, the entire radial thickness with graded material distribution is not important in order to meet the requirements of a cutter or a grinder. Using only a thin FGM coating at the outer surface can suffice to meet the requirements of an application and is a more feasible approach from the technical and manufacturing points of view. However, to design such a cutter with an FGM coating at the outer surface having desired performance, it is a prerequisite to understand and quantify the thermoelastic characteristics of such a circular disk with an FGM coating at the outer surface.

Therefore, the present study focuses on the analysis of thermoelastic characteristics of a rotating circular disk with an FGM coating at the outer surface. The finite element model developed in our previous study [20] is modified for the present model of the problem and used to obtain the solution of stress, displacement, and strain components as a function of temperature distribution, angular speed of the disk and FGM coating thickness.

2. Modeling of the Problem

The outer surface and its adjoining region of a circular cutter or a grinding disk should be made of such a material that can resist wear and high temperature while the inner region should be tougher and thermally conductive to absorb torsional energy and facilitate cooling. A simple bi-metallic disk made of ceramic at the outer surface and metal at the inner region can meet the above requirements. However, the sharp interface between ceramic and metal induces the problem of delamination and ultimate failure of the disk. To get rid of this problem, the disk can be modeled as consisting of a metallic material with an FGM coating at its outer surface as shown in Fig. 1. Shown in Fig. 1 is the polar coordinate system r–θ and a

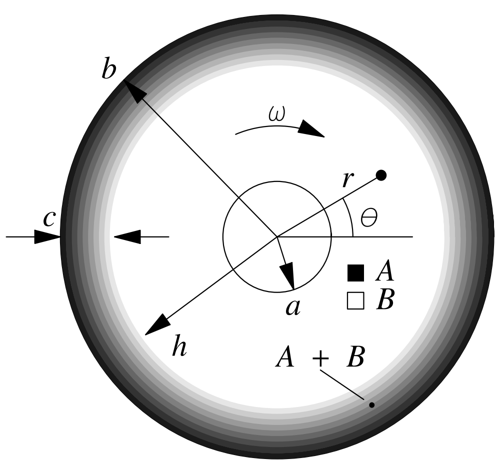

Figure 1. Analytical model of the disk problem.

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concentric hole of radius a. The outer radius of the disk is represented by b. The region $(h-a)$ consists of a homogeneous material B while the coating of thickness $c=(b-h)$ consists of the A/B FGM. The black and white colors are used to denote the materials A and B, respectively. The FGM coating has the 100% material A at $r=b$ and 100% material B at $r=h$ with continuous variation of material distribution between b and h. Further, it is assumed that the disk is rotating at N rpm that corresponds to the angular speed $\omega = 2\pi N/60$ rad/s.

The Young's modulus E, CTE α, and density ρ are assumed to vary exponentially with r only over the coating region c while the Poisson's ratio ν remains constant throughout the entire disk. The exponential variation of the above properties are given by

$$E = E_0 \mathrm{e}^{\beta r}, \tag{1a}$$

$$\alpha = \alpha_0 \mathrm{e}^{\gamma r}, \tag{1b}$$

$$\rho = \rho_0 \mathrm{e}^{\mu r}. \tag{1c}$$

From the fact that the materials properties E, α, and ρ assume the values of the corresponding properties of the materials A and B at $r=b$ and h, respectively, the constants of equation (1) can be determined as

$$E_0 = E_{\mathrm{B}} \mathrm{e}^{-\beta h}, \tag{2a}$$

$$\alpha_0 = \alpha_{\mathrm{B}} \mathrm{e}^{-\gamma h}, \tag{2b}$$

$$\rho_0 = \rho_{\mathrm{B}} \mathrm{e}^{-\mu h}, \tag{2c}$$

$$\beta = \frac{1}{b-h} \ln\left(\frac{E_{\mathrm{A}}}{E_{\mathrm{B}}}\right), \tag{3a}$$

$$\gamma = \frac{1}{b-h} \ln\left(\frac{\alpha_{\mathrm{A}}}{\alpha_{\mathrm{B}}}\right), \tag{3b}$$

$$\mu = \frac{1}{b-h} \ln\left(\frac{\rho_{\mathrm{A}}}{\rho_{\mathrm{B}}}\right). \tag{3c}$$

Equations (1)–(3) simulate the material properties of the FGM coating region $h \leqslant r \leqslant b$ of the disk. However, the same expressions can be used for the determination of the properties of the homogeneous region $a \leqslant r \leqslant h$ of the disk by setting the parameters β, γ and μ to zero.

As the CTE of the disk is nonuniform, an incompatible eigenstrain [21] is developed as a result of change in temperature. For the present model of the problem, the eigenstrain is the function of r only, which can be given by

$$\varepsilon^* = \alpha(r) T(r), \tag{4}$$

where $T(r)$ is the distribution of temperature along the radial direction of the disk.

3. Formulation of the Problem

The present study is based on the thermoelastic formulations developed in our previous study [20] associated with an FGM disk in which the material properties are assumed to vary throughout entire radial thickness of the disk. Thus, this section is intended to give a concise description of the thermoelastic formulations of our previous model followed by the discussion of how these formulations can be applied to the present model of the problem.

4. FGM Disk ($h = a$)

Let us consider Fig. 1 and imagine that the parameter h is equal to the inner radius a of the disk. This indicates an FGM disk with the material distribution as well as material properties varying in the entire radial thickness of the disk. For such a disk, making use of 2-D plane stress equilibrium equation in polar coordinate, strain–stress relations, strain–displacement relations and equations (1)–(4) yields [20]

$$\begin{aligned}\frac{\mathrm{d}^2F}{\mathrm{d}r^2} &+ \left(\frac{1}{r} - \beta\right)\frac{\mathrm{d}F}{\mathrm{d}r} + \frac{1}{r}\left(\beta\nu - \frac{1}{r}\right)F \\ &= \rho\omega^2 r(\beta r - \mu r - \nu - 3) - E\alpha\left(\gamma T + \frac{\mathrm{d}T}{\mathrm{d}r}\right).\end{aligned} \tag{5}$$

Once equation (5) is solved for F, the components of stress, strain and displacement are obtained from Ref. [20] as

$$\sigma_{\mathrm{r}} = \frac{F}{r}, \tag{6a}$$

$$\sigma_\theta = \frac{\mathrm{d}F}{\mathrm{d}r} + \rho\omega^2 r^2, \tag{6b}$$

$$\varepsilon_{\mathrm{r}} = \frac{1}{E}\left(\frac{F}{r} - \nu\frac{\mathrm{d}F}{\mathrm{d}r}\right) - \frac{\nu\rho}{E}\omega^2 r^2 + \varepsilon^*, \tag{7a}$$

$$\varepsilon_\theta = \frac{1}{E}\left(\frac{\mathrm{d}F}{\mathrm{d}r} - \frac{\nu F}{r}\right) + \frac{\rho\omega^2 r^2}{E} + \varepsilon^*, \tag{7b}$$

$$u_{\mathrm{r}} = \varepsilon_\theta r, \tag{8}$$

where σ_{r} and σ_θ are the radial and circumferential stress components, ε_{r} and ε_θ are the radial and circumferential strain components, and u_{r} is the radial displacement.

As the analytical solution of equation (5) is not realistic, a finite element model is developed to obtain its solution. It is noted that it is a one-dimensional problem, i.e., all the parameters of interest are a function of r only. Thus, only the radial domain of the disk $\Omega = (a, b)$ is divided into S number of subdomains $\Omega^e = (r_e, r_{e+1})$, where the element number $e = 1, 2, \ldots, S$. Then, following the variational approach and Ritz method, equation (5) is reduced to its corresponding finite element form as [20]

$$\sum_{j=1}^{2} K_{ij}^e F_j^e = L_i^e, \tag{9}$$

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where

$$K_{ij}^e = B(\phi_i^e, \phi_j^e), \tag{10a}$$

$$L_i^e = l(\phi_i^e), \tag{10b}$$

$$F = \sum_{j=1}^{2} F_j^e \phi_j^e, \tag{10c}$$

$$\begin{aligned} B(w, F) = {} & \int_{\mathrm{r}_e}^{r_{e+1}} \frac{\mathrm{d}w}{\mathrm{d}r}\frac{\mathrm{d}F}{\mathrm{d}r}\,\mathrm{d}r \\ & - \int_{\mathrm{r}_e}^{r_{e+1}} \left(\frac{1}{r} - \beta\right) w \frac{\mathrm{d}F}{\mathrm{d}r}\,\mathrm{d}r - \int_{\mathrm{r}_e}^{r_{e+1}} \frac{1}{r}\left(\beta\nu - \frac{1}{r}\right) wF\,\mathrm{d}r, \end{aligned} \tag{11a}$$

$$l(w) = -\int_{\mathrm{r}_e}^{r_{e+1}} wf(r)\,\mathrm{d}r + w(r_{e+1})\frac{\mathrm{d}F}{\mathrm{d}r}(r_{e+1}) - w(r_e)\frac{\mathrm{d}F}{\mathrm{d}r}(r_e), \tag{11b}$$

$$f(r) = \rho\omega^2 r(\beta r - \mu r - \nu - 3) - E\alpha\left[\gamma T + \frac{\mathrm{d}T}{\mathrm{d}r}\right], \tag{11c}$$

$$\phi_1^e = \frac{r_{e+1} - r}{r_{e+1} - r_e}, \tag{12a}$$

$$\phi_2^e = \frac{r - r_e}{r_{e+1} - r_e}. \tag{12b}$$

In deriving equation (9), the following boundary conditions were used:

(i) $r = a$, $\sigma_{\mathrm{r}} = 0$, i.e., $F(a) = 0$;

(ii) $r = b$, $\sigma_{\mathrm{r}} = 0$, i.e., $F(b) = 0$.

Equation (9) is a system of linear algebraic equations which is used to form a global system of equations from the continuity condition of $F_2^e = F_1^{e+1}$. Once the values of F_j^e are known at the global node points, the components of stress, strain, and displacement for each element can be determined from [20]

$$\sigma_{\mathrm{r}}^e = \frac{1}{r}\sum_{j=1}^{2} F_j^e \phi_j^e, \tag{13a}$$

$$\sigma_\theta^e = \sum_{j=1}^{2} F_j^e \frac{\mathrm{d}\phi_j^e}{\mathrm{d}r} + \rho\omega^2 r^2, \tag{13b}$$

$$\varepsilon_{\mathrm{r}}^e = \frac{1}{E}\sum_{j=1}^{2}\left[\frac{F_j^e\phi_j^e}{r} - \nu F_j^e \frac{\mathrm{d}\phi_j^e}{\mathrm{d}r}\right] - \frac{\nu\rho\omega^2 r^2}{E} + \varepsilon^*, \tag{14a}$$

$$\varepsilon_\theta^r = \frac{1}{E}\sum_{j=1}^{2}\left[F_j^e \frac{\mathrm{d}\phi_j^e}{\mathrm{d}r} - \frac{\nu}{r} F_j^e \phi_j^e\right] + \frac{\nu\rho\omega^2 r^2}{E} + \varepsilon^*, \tag{14b}$$

$$u_{\mathrm{r}}^{e} = \frac{r}{E}\sum_{j=1}^{2}\left[F_j^e \frac{\mathrm{d}\phi_j^e}{\mathrm{d}r} - \frac{\nu}{r}F_j^e\phi_j^e\right] + \frac{\nu\rho\omega^2 r^3}{E} + r\varepsilon^*. \tag{15}$$

4.1. Disk with FGM Coating ($h > a$)

Figure 1 shows a disk with an FGM coating at the outer surface. As mentioned earlier, the parameter h is used to denote the radius of the homogeneous part of the disk. The difference between the FGM disk ($h = a$) and the disk with an FGM coating ($h > a$) is that the FGM disk ($h = a$) has the material gradation throughout the entire radial thickness ($b - a$) while the disk with an FGM coating ($h > a$) has the gradation of material only in the FGM coating region $c = (b - h)$, where $h > a$. The formulations outlined in the preceding article 3.1. for an FGM disk can be used for the present case of the disk with an FGM coating by considering the fact that the present model of the disk has two regions: the homogeneous region ($h - a$) with constant material properties and the FGM region ($b - h$) with continuously varying material properties. The two regions can be governed by the same formulations outlined above except the values of the three parameters β, γ and μ. For the FGM coating region ($b - h$), these parameters are determined from equation (3). On the other hand, these parameters are set to zero for the homogeneous region ($h - a$). To solve the global system of algebraic equations derived from equation (9) and the continuity condition $F_2^e = F_1^{e+1}$, one can write a simple program in which β, γ and μ can be determined from equation (3) if r corresponds to a point in the coating region ($b - h$) and set to zero value if r falls in the homogeneous region ($h - a$).

5. Numerical Results and Discussion

The finite element model developed in this study for the analysis of thermoelastic characteristics of a rotating disk with an FGM coating is applied to an Al disk with an Al_2O_3/Al FGM coating at the outer surface. The constituent materials A and B shown in Fig. 1 correspond to Al_2O_3 and Al, respectively. The mechanical and thermal properties of Al and Al_2O_3 are shown in Table 1. The range of variation of Poisson's ratio is insignificant and has a negligible effect on the thermoelastic characteristics. Thus, the Poisson's ratio is assumed to have a constant value of 0.3 throughout the disk.

Table 1.
Mechanical and thermal properties of Al and Al_2O_3

Materials	Properties		
	Young's modulus (MPa)	Coefficient of thermal expansion (/°C)	Density (g/cm^3)
Al	71	23.1×10^{-6}	2.70
Al_2O_3	380	8.0×10^{-6}	0.96

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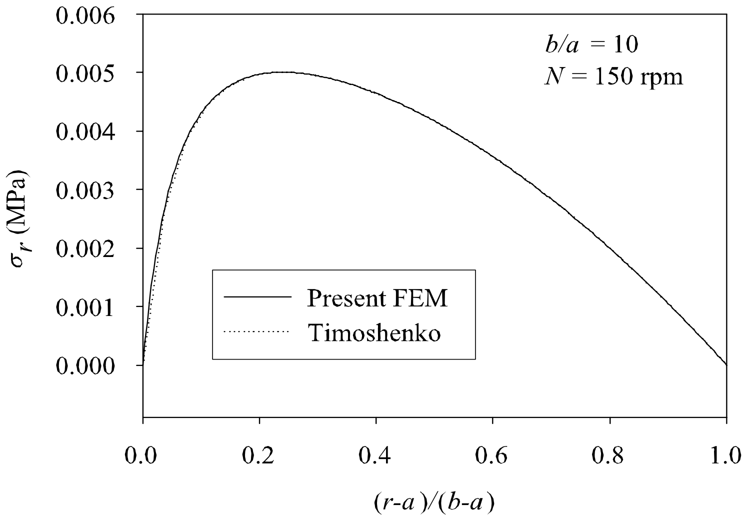
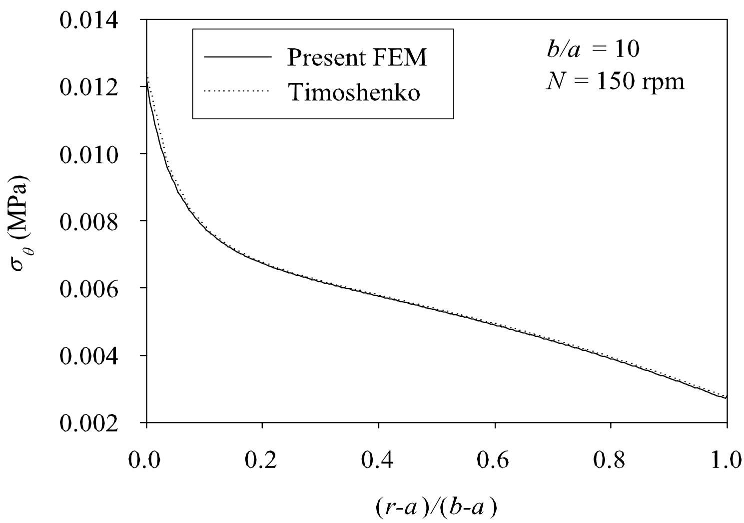
The global system of algebraic equations is solved by writing a program in MATHEMATICA 5.1. The validity of the present finite element model is verified for a homogeneous disk consisting entirely of Al. The disk is assumed to be rotating at $N = 150$ rpm and free from any thermal load. For $a = 15$ mm and $b/a = 10$, the homogeneous disk problem under rotation only is solved by the present finite element method and the results are compared with the analytical solution obtained by Timoshenko and Goodier [22] as shown in Figs 2 and 3.



(a)



(b)

Figure 2. Comparison of (a) radial and (b) circumferential stress in a homogeneous circular disk under rotation.

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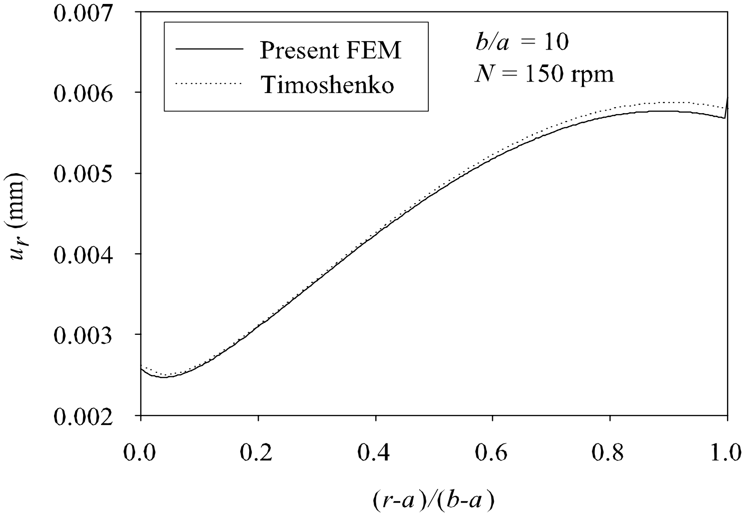

Figure 3. Comparison of radial displacement in a homogeneous circular disk under rotation.

Note that to solve the homogeneous disk problem under rotation only, the parameters β, γ, μ and ε^* are set to zero for the entire radial thickness of the disk. It is observed from Figs 2 and 3 that the finite element results agree well with the analytical results obtained by Timoshenko and Goodier [22]. The maximum difference between the present finite element and analytical results is 2.5%, which takes place in the case of the displacement at the outer surface of the disk. This small discrepancy in the results occurs due to the inherent numerical error associated with the finite element method. Thus, it is verified that the present finite element method is reliable in producing the results with high accuracy.

Prior to the calculation of the final results, the effect of the element size of the meshed disk on the results is investigated. Note that the disk is meshed in the radial direction only with one-dimensional elements as all the parameters are the function of r only. Further, the elements have uniform distribution and equal size. The test of convergence as a function of the element size is carried out only for the parameter F. As all the parameters of interest, namely stress, strain and displacement, are computed from F, the convergence of F ensures the convergence of the stress, strain and displacement. Figure 4 exhibits the effect of the element size on the parameter F. It is found that the values of F converge very well for the element size of 5 mm. However, for further accuracy of the results, the size of the elements is chosen as 1 mm in calculating the subsequent final results of stress, strain and displacement.

To investigate the effect of temperature distribution profile on the thermoelastic characteristics of the disk, three different distributions of temperature as shown in Fig. 5 are prescribed along the radial direction of the disk. The distributions of tem-

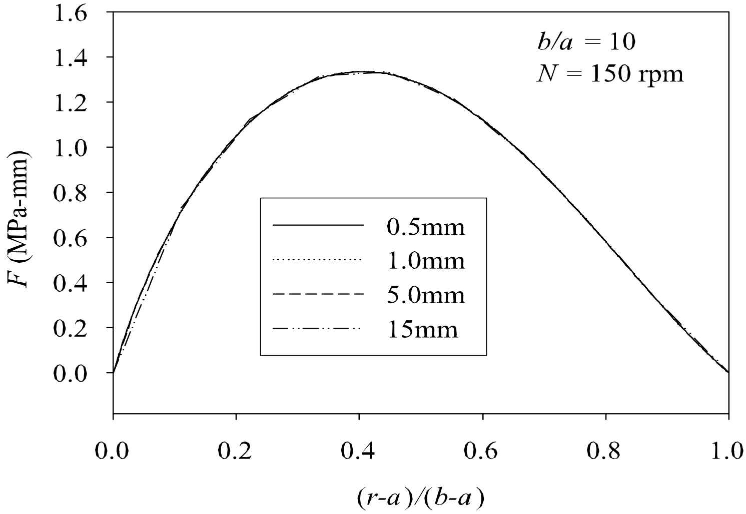

Figure 4. Effect of elemental length on the parameter F.

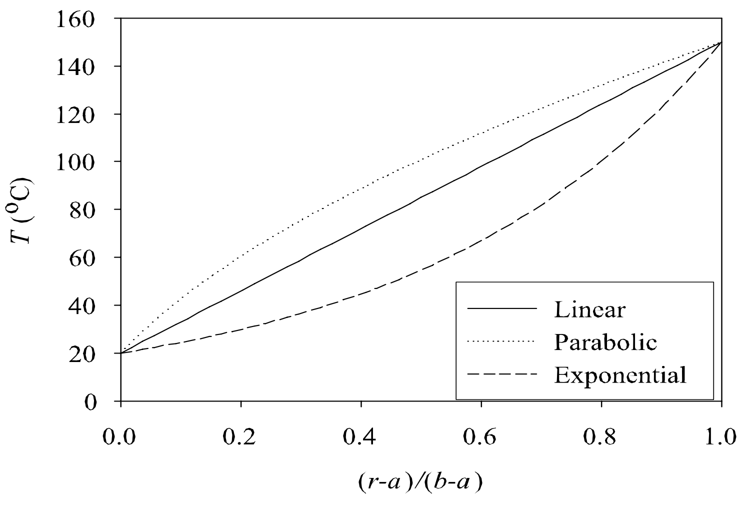

Figure 5. Temperature distributions along the radial direction.

perature are intuitively prescribed to mimic the similar temperature distributions that may be encountered in an actual grinding or cutting disk. A grinding or cutting disk, in fact, experiences a higher temperature at the grinding or cutting (outer) surface from which the temperature will gradually fall toward the inner surface following any one of the similar profiles shown in Fig. 5, i.e., linear, exponential or parabolic. The outer surface (cutting or grinding surface) temperature is taken as 150°C while the inner surface temperature is set to room temperature (20°C).

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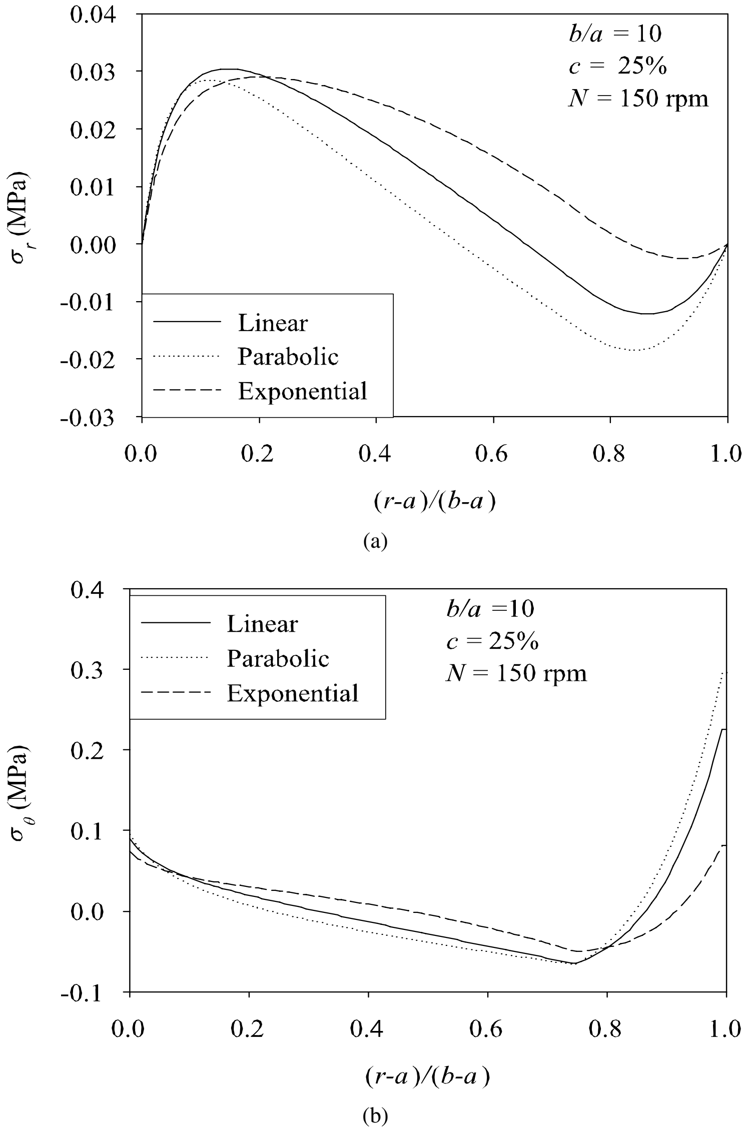
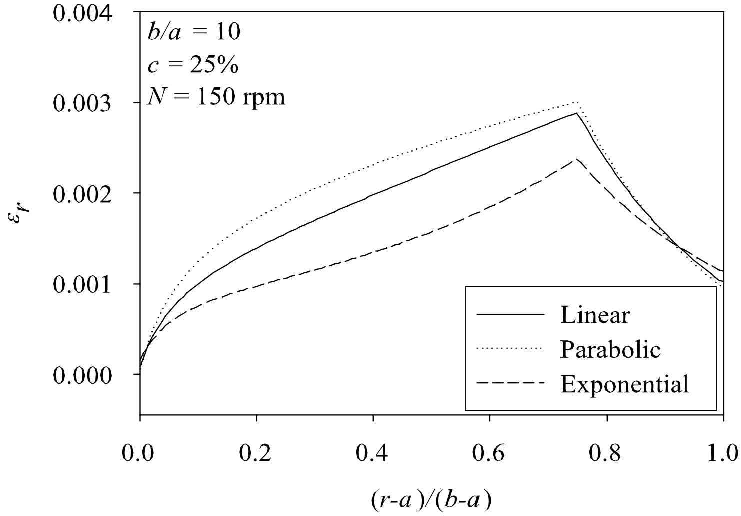
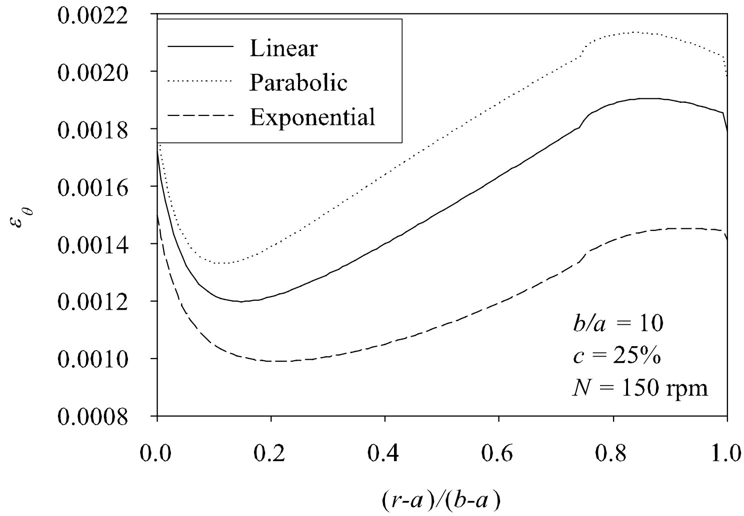


Figure 6. Effect of temperature distribution on (a) radial and (b) circumferential stress.

For these prescribed temperature distributions and $N = 150$ rpm, $a = 15$ mm, $b/a = 10$, coating thickness $c = 25\%$, the computed distributions of stress, strain and displacement are shown in Figs 6, 7 and 8, respectively. For any temperature distribution, the radial stress as shown in Fig. 6a is zero at the inner and outer surfaces of the disk, which satisfies the boundary condition of the problem. The radial stress is tensile over the inner region of the disk and compressive over the outer region of the disk. This type of stress distribution is in fact desirable as the outer



(a)



(b)

Figure 7. Effect of temperature distribution on (a) radial and (b) circumferential strain.

region of the disk is ceramic (Al_2O_3) rich, which has better compressive strength. On the other hand, the inner region consists of purely metallic material (Al), which has better tensile strength. Further, it is noted that the linear temperature distribution develops the maximum tensile stress at the inner region while the parabolic temperature distribution develops the maximum magnitude of compressive stress at the outer region of the disk. Figure 6b shows the circumferential stress distribution along the radial direction of the disk. Only the small region around the

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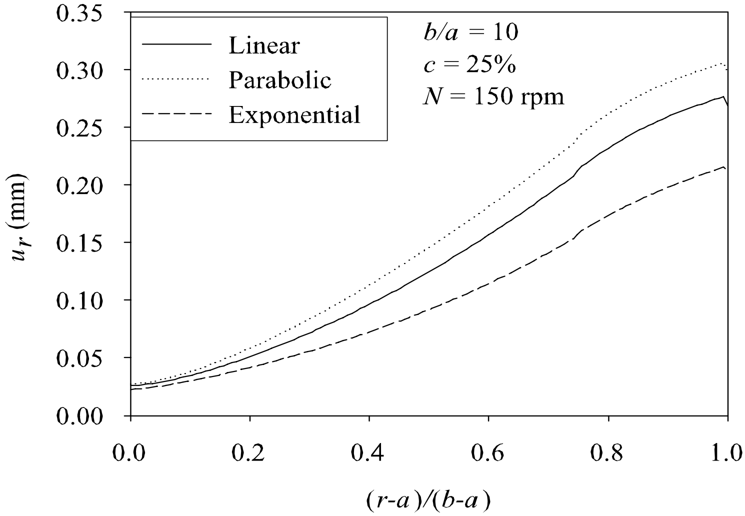

Figure 8. Effect of temperature distribution on displacement.

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interface between the FGM coating and the homogeneous part of the disk suffers from the compressive circumferential stress. The inner and the outer regions including the boundaries suffer from the tensile circumferential stress. Although the temperature distribution profiles shown in Fig. 5 have little effect on the circumferential stress over the inner region of the disk, their effect on the stress is significant at and near the outer surface of the disk. The parabolic temperature distribution induces the maximum circumferential stress over this region. The effect of temperature distribution on the radial and circumferential strain components are displayed in Figs 7a and 7b, respectively. Over the entire radial thickness of the disk, both the strain components have the maximum magnitude for the parabolic temperature distribution while their magnitude is the minimum for the exponential temperature distribution. Further, the distribution of both the strain components has a point of inflection at the interface between the FGM coating and the homogenous part of the disk. The effect of temperature distribution on the displacement is exhibited in Fig. 8. For all the temperature distributions, the displacement is the minimum at the inner surface and it monotonically increases toward the outer surface of the disk. Like strain components, the displacement is also the maximum for the parabolic temperature distribution and minimum for the exponential temperature distribution throughout the entire radial thickness of the disk.

To investigate the effects of angular speed of the disk on the thermoelastic characteristics, the linear temperature distribution profile of Fig. 5 is considered and different components of stress, strain and displacement are evaluated and presented in Figs 9, 10 and 11, respectively. All the results correspond to $b/a = 10$ and $c = 15\%$. Four different angular speeds, namely $N = 200$, 600, 1000 and 1500 rpm, are considered to understand how the thermoelastic characteristics are influenced

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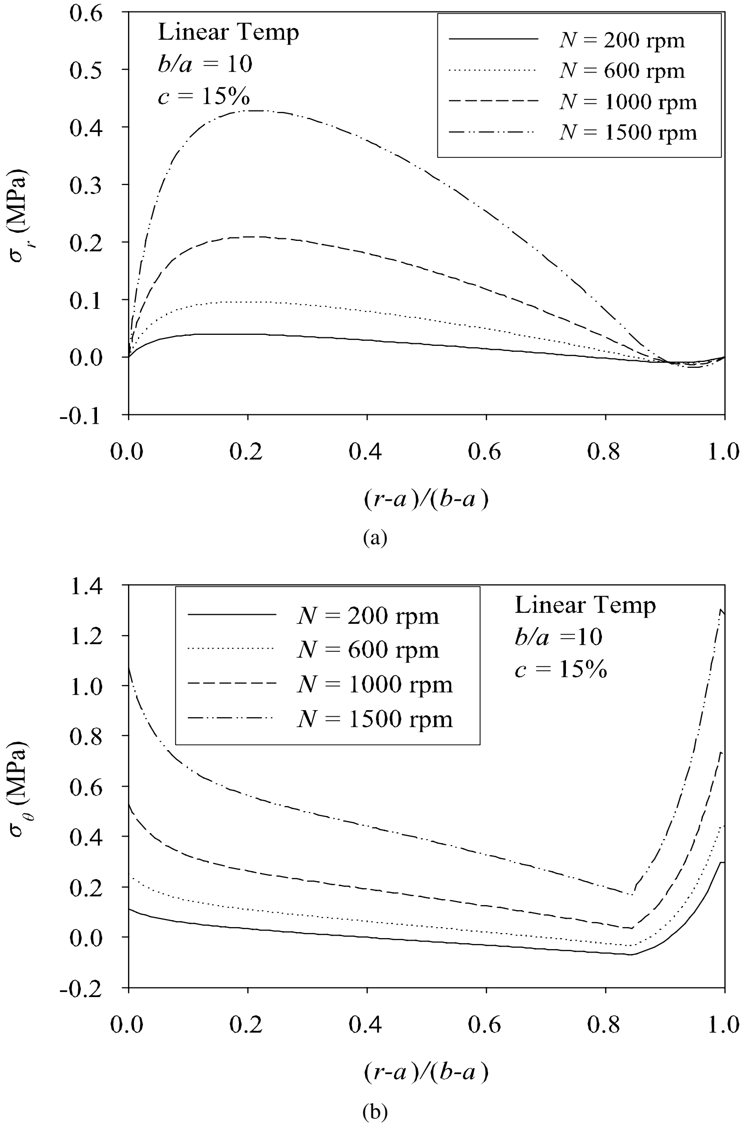

Figure 9. Effect of angular speed on (a) radial and (b) circumferential stress.

by the angular speed. The inertial force is proportional to the square of the angular speed of the disk. Further, the stress, strain and displacement are, in general, proportional to the force acting on the disk. Thus, the magnitude of the stress, strain and displacement generally increases with the increase of the angular speed as shown in Figs 9–11. The component of the radial strain (Fig. 1a) shows only a little exception of this general phenomenon. At and near the outer surface of the disk, the magnitude of the radial strain varies inversely with the angular speed. Further, the

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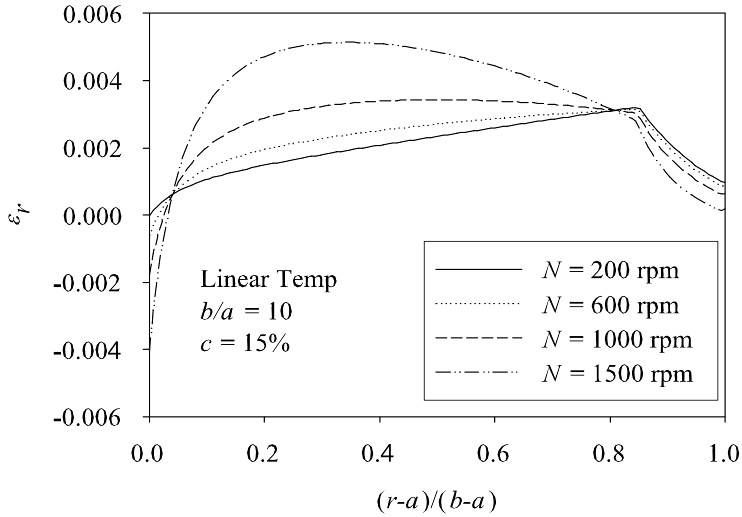
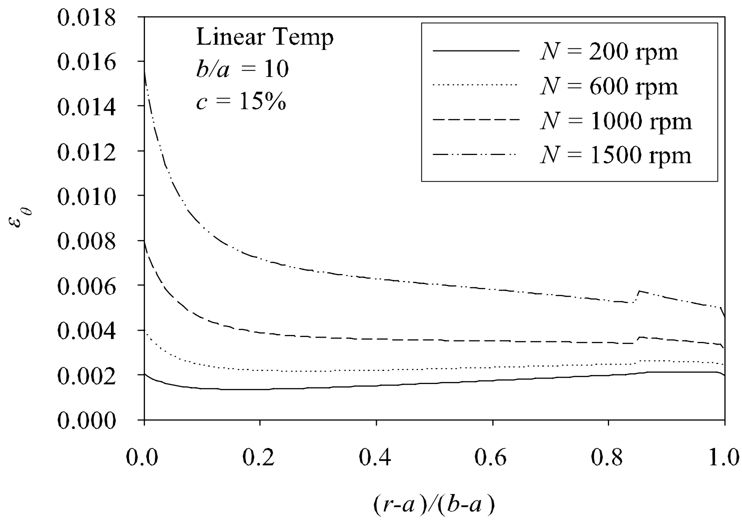


(a)



(b)

Figure 10. Effect of angular speed on (a) radial and (b) circumferential strain.

variation of the magnitude of the stress, strain and displacement with the angular speed is distinct over the entire radial thickness of the disk, except for the case of the radial stress. As seen from Fig. 9a, the radial stress changes little with the variation of the angular speed over the outer region (about 10% of the radial thickness) of the disk.

The effects of FGM coating thickness on the components of stress, strain and displacement are illustrated in Figs 12, 13 and 14, respectively. The results corre-

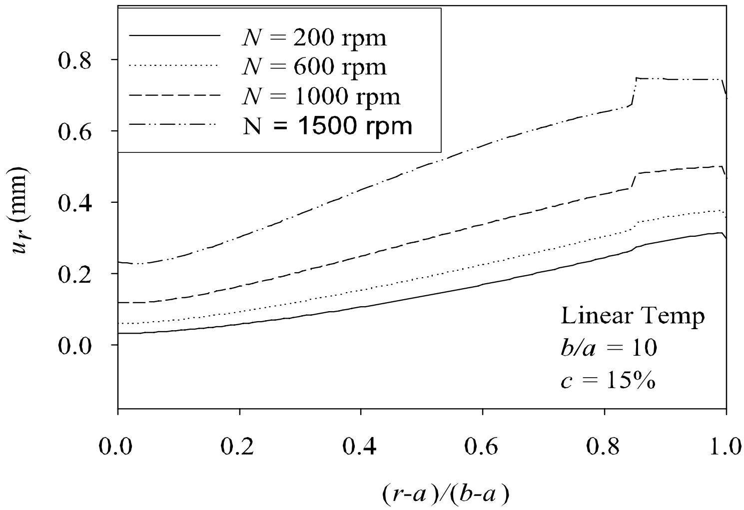


Figure 11. Effect of angular speed on displacement.

spond to $b/a = 10$, $N = 150$ rpm, and the linear temperature distribution of Fig. 5. The results are presented for the FGM coating thickness of $c = 20, 15, 10$, and 5% of the total radial thickness of the disk. Figure 12a shows that, over the major part of the inner region of the disk, the radial stress is tensile and its magnitude increases with the decrease of the coating thickness. Only over the outer smaller region of the disk, does the radial stress show the reverse trend, i.e., the radial stress is compressive over this region and its magnitude rises as the coating thickness increases. The characteristics of circumferential stress as a function of the FGM coating thickness are displayed in Fig. 12b. The circumferential stress, like radial stress, increases as the coating thickness decreases over the inner region of the disk. At the outer surface, the same phenomenon is observed. However, the feature of this stress is different over the region at and near the interface between the FGM coating and the homogeneous part of the disk. The radial and circumferential strain components as shown in Figs 13a and 13b, respectively, have higher magnitude over the entire radial thickness of the disk for the lower value of the coating thickness. The distribution of displacement component as a function of the coating thickness as shown in Fig. 14 has also the similar characteristics as those of strain components. The magnitude of displacement increases with the decrease of the coating thickness.

6. Conclusions

Based on our previous study, a finite element model has been developed for the analysis of thermoelastic characteristics of a rotating circular disk with an FGM coating at the outer surface of the disk. The finite element model of the present study is more generalized as it can be applied to

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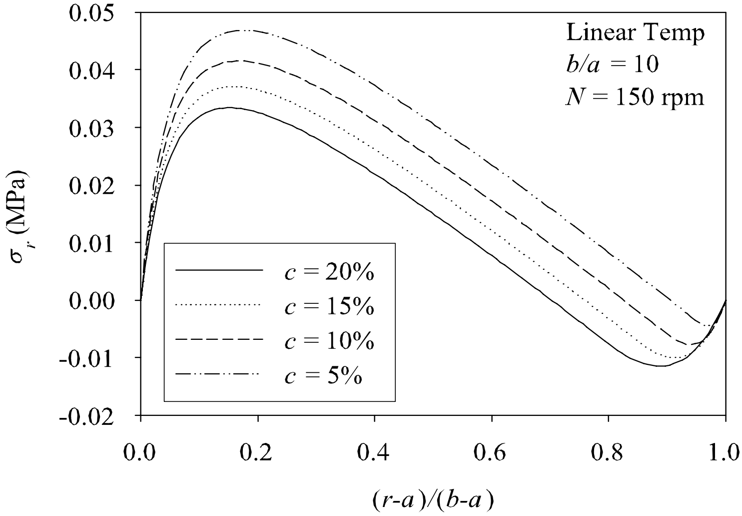
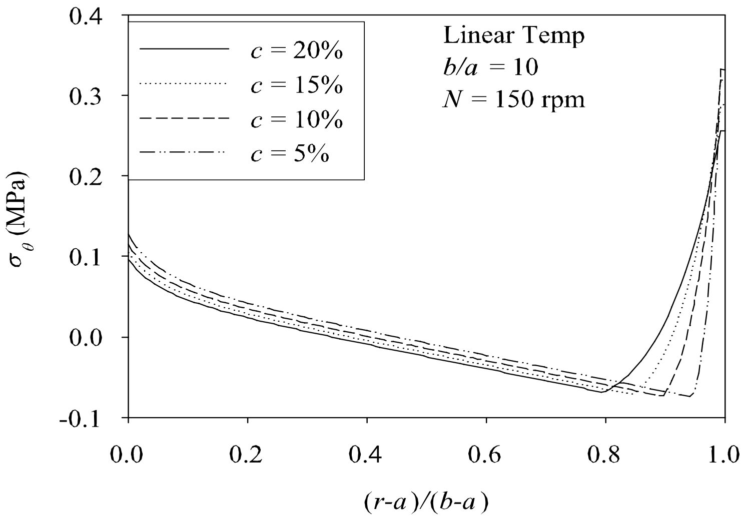


(a)



(b)

Figure 12. Effect of FGM coating thickness on (a) radial and (b) circumferential stress.

(i) an FGM disk having material gradation throughout the entire radial thickness of the disk by setting the parameter $h = a$;

(ii) a disk with an FGM coating by setting $h > a$;

(iii) a homogeneous disk by setting the parameters $\beta = \gamma = \mu = 0$.

The model is verified for a homogeneous disk whereby the reliability of the model is established. It is demonstrated for an Al disk with an Al_2O_3/Al

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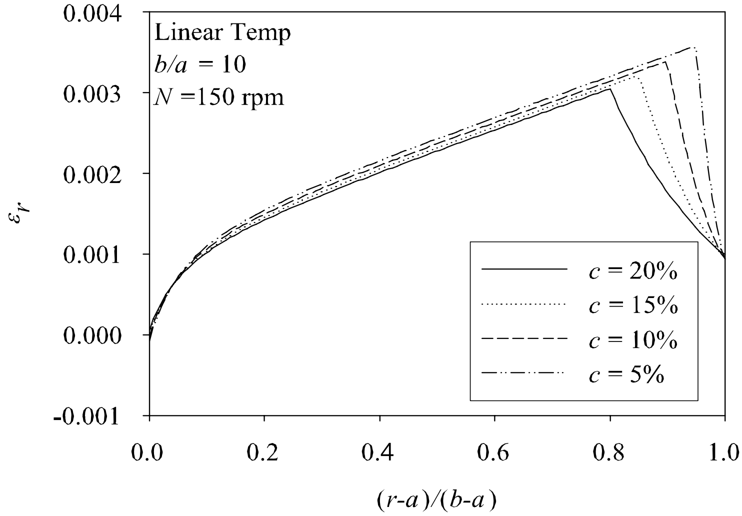
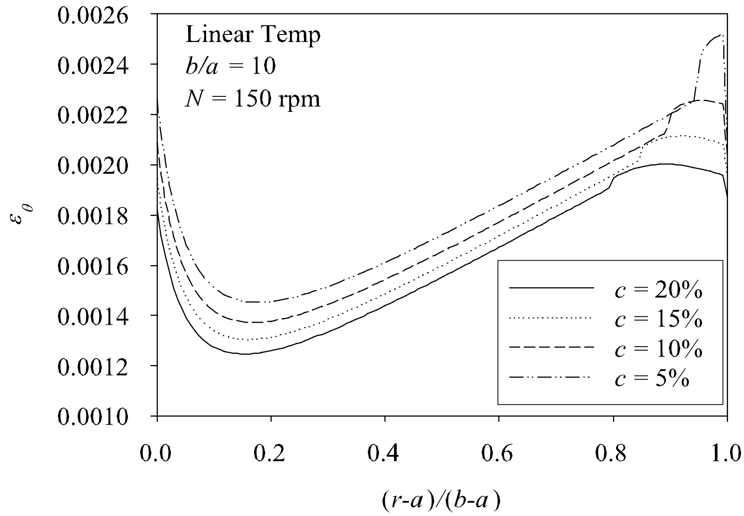


(a)



(b)

Figure 13. Effect of FGM coating thickness on (a) radial and (b) circumferential strain.

FGM coating at the outer surface of the disk. The numerical results show that the temperature distribution profile, angular speed of the disk, and FGM coating thickness are the crucial parameters to characterize the thermoelastic behavior of the disk. By controlling these parameters, a desired characteristic of the disk can be ensured. Further, the numerical results presented in this study are helpful in designing with an actual cutter or grinder with an FGM coating.

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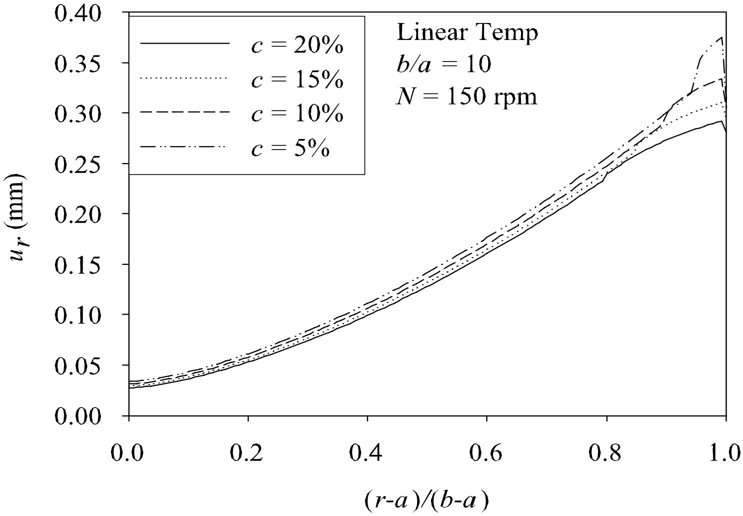

Figure 14. Effect of FGM coating thickness on displacement.

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Acknowledgement

This work was supported by the Korea Research Foundation (KRF) grant funded by the Korea government (MEST) (No. KRF 2009-0076450). Also, the partial support of the Brain Korea 21 Project Corps. of the second stage is gratefully acknowledged.

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